Proceedings of the
International Conference on Mechanical Engineering 2011
(ICME2011) 18- 20 December 2011, Dhaka, Bangladesh

ICME11-TH-038

PRESSURE-DRIVEN FLOW INSTABILITY WITH CONVECTIVE HEAT TRANSFER THROUGH A CURVED CHANNEL WITH STRONG CURVATURE

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ABSTRACT

Pressure-driven flow instability with convective heat transfer through a curved square channel with strong curvature has been performed numerically by using a spectral method and covering a wide range of the Dean number. Numerical calculations are carried out over the Dean number $100 \le Dn \le 6500$ for the curvature $\delta = 0.5$. A temperature difference is applied across the vertical sidewalls for the Grashof number $Gr = 100$, where the outer wall is heated and the inner one cooled. After a compressive survey over the parametric ranges, two branches of asymmetric steady solutions with two- and four-vortex solutions are obtained by the Newton-Raphson iteration method. Then, in order to investigate the non-linear behavior of the unsteady solutions, time evolution calculations as well as power spectrum of the solutions are obtained, and it is found that the steady-state flow turns into chaotic flow through periodic and multi-periodic flows if Dn is increased up to 5835. For large Dean numbers $(\mathrm{Dn} > 5835)$, however, the unsteady flow undergoes through various flow instabilities if, Dn is increased gradually. It is found that the maximum axial velocity is shifted near the outer wall of the duct if Dn is increased. Transition of the unsteady solutions is determined by the power spectrum of the solution.

Keywords: Curved Square Duct, Secondary Flow, Time Evolution, Periodic Solution, Heat Transfer.

1. INTRODUCTION

The flow through curved a duct shows physically interesting features under the action of centrifugal force caused by the curvature of the duct. Dean (1927) first formulated the problem in mathematical terms under the fully developed flow and showed the existence of a pair of counter rotating vortices in a curved pipe. The readers are referred to Berger *et al*. (1983) and Yanase *et al.* (2005) for some reviews on curved duct flows.

Considering the non-linear nature of the Navier-Stokes equation, the existence of multiple solutions does not come as a surprise. An early complete bifurcation study of two-dimensional (2-D) flow through a curved duct of square cross section was conducted by Winters (1987). Very recently, Mondal *et al*. (2007) performed comprehensive numerical study on fully developed bifurcation structure and stability of two-dimensional (2D) flow through a curved duct with square cross section and found a close relationship between the unsteady solutions and the bifurcation diagram of steady solutions. The flow through a curved duct with differentially heated vertical sidewalls has another aspect because secondary flows promote fluid mixing and heat transfer in the fluid (Yanase *et al*., 2005). Recently, Mondal *et al*. (2008) performed numerical investigations of non-isothermal flows through a curved duct with square cross section, where they studied the flow characteristics with the effects of secondary flows on convective heat transfer.

One of the interesting phenomena of the flow through curved duct is the bifurcation of the flow because generally there exist many steady solutions due to channel curvature. Recently, Mondal *et al*. (2009, 2010) performed numerical prediction of the unsteady solutions through a stationary curved square duct flow for both the isothermal and non-isothermal flows. They showed that periodic solutions turn into chaotic solution through a multi-periodic solution, if the Dean number is increased no matter what the curvature is. They also showed that the chaotic solution becomes weak for small Dean number, while the chaotic solution becomes strong for large Dean number. The paper is an attempt to fill up the gap with a view to study the non-linear nature of the unsteady solutions for strong curvature and large pressure gradient, because this type of flow of often encountered in engineering applications.

In the present study, a numerical result is presented for the fully developed two-dimensional flow of viscous incompressible fluid through a curved square duct of strong curvature. Another objective of the present study is to investigate the unsteady flow behavior in the presence of buoyancy force.

2. MATHEMATICAL FORMULATION

Consider a viscous incompressible fluid streaming through a curved square duct of curvature 0.5. The coordinate system with relevant notations is shown in Fig. 1. It assumed that the flow is uniform in the z-direction and that the outer wall of the duct is heated while the inner wall cooled. *u*, *v* and *w* are the velocity components in the *x*-, *y*- and *z*-directions, respectively. The variables are non-dimensionalized by using the representative length and the representative velocity.

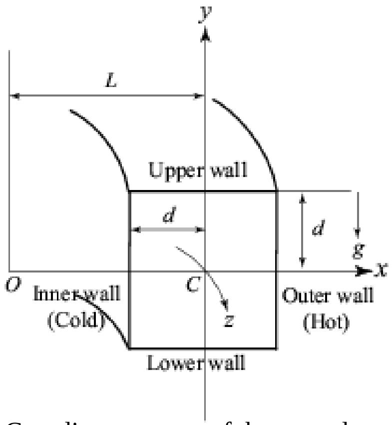


Fig 1. Coordinate system of the curved square duct.

The sectional stream function $\psi(x, y)$ is introduced in the *x*- and *y*- directions as

$$u = \frac{1}{1+\delta x}\frac{\partial \psi}{\partial y}, \qquad v = \frac{1}{1+\delta x}\frac{\partial \psi}{\partial x} \tag{1}$$

Then the basic equations for *w*, ψ and *T* are derived from the Navier-Stockes equations as

$$(1+\delta x)\frac{\partial w}{\partial t} + \frac{\partial(w,\psi)}{\partial(x,y)} \quad Dn + \frac{\delta^2 w}{1+\delta x} = (1+\delta x)\Delta_2 w \quad \frac{\delta}{(1+\delta x)}\frac{\partial \psi}{\partial y} w + \delta \frac{\partial w}{\partial x} \tag{2}$$

$$\Delta_2 \quad \frac{\delta}{1+\delta x}\frac{\partial}{\partial x} \quad \frac{\partial \psi}{\partial t} = \quad \frac{1}{(1+\delta x)}\frac{\partial(\Delta_2\psi,\psi)}{\partial(x,y)} + \frac{\delta}{(1+\delta x)^2} \quad \frac{\partial \psi}{\partial y} \quad 2\Delta_2\psi \quad \frac{3\delta}{1+\delta x}\frac{\partial \psi}{\partial x} + \frac{\partial^2 \psi}{\partial x^2} \quad \frac{\partial \psi}{\partial x}\frac{\partial^2 \psi}{\partial x \partial y} \quad + \frac{\delta}{(1+\delta x)^2} \quad 3\delta \frac{\partial^2 \psi}{\partial x^2} \quad \frac{3\delta^2}{1+\delta x}\frac{\partial \psi}{\partial x} \quad \frac{2\delta}{1+\delta x}\frac{\partial}{\partial x}\Delta_2\psi + w\frac{\partial \psi}{\partial y} \quad G_r\,(1+\delta x)\frac{\partial T}{\partial x} + \Delta_2^2\psi \tag{3}$$

$$\frac{\partial T}{\partial t} + \frac{1}{(1+\delta x)}\frac{\partial(T,\psi)}{\partial(x,y)} = \frac{1}{Pr} \quad \Delta_2 T + \frac{\delta}{1+\delta x}\frac{\partial T}{\partial x} \tag{4}$$

where $\Delta_2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad \frac{\partial(f,g)}{\partial(x,y)} \equiv \frac{\partial f \partial g}{\partial x \partial y} \quad \frac{\partial f \partial g}{\partial y \partial x}$

The non-dimensional parameters Dn, the Dean number, Gr, the Grashof number and Pr, the Prandtl number, which appear in equations (2) to (4) are defined as:

$$Dn = \frac{Gl^3}{\mu\upsilon}\sqrt{\frac{2l}{L}}, \quad Gr = \frac{\beta g \Delta T\, l^3}{\upsilon^2}, \quad Pr = \frac{\upsilon}{\kappa}.$$

Here, *l* is the aspect ratio defined as $l = \frac{h}{d}$ and δ is the curvature ($\delta = 0.5$). Here *l* = 1 (square duct).

The boundary conditions for w and ψ are used as

$$w(\pm 1, y) = w(x, \pm 1) = \psi(\pm 1, y) = \psi(x, \pm 1) = \frac{\partial \psi}{\partial x}(\pm 1, y) = \frac{\partial \psi}{\partial y}(x, \pm 1) = 0 \tag{5}$$

and the temperature T is assumed to be constant on the walls as:

$$T(1, y) = 1, \quad T(\quad 1, y) = \quad 1, T(x, \pm 1) = x \tag{6}$$

3. NUMERICAL CALCULATION

In order to obtain the numerical solutions, spectral method is used. The main objective of the method is to use the expansion of the polynomial functions that is the variables are expanded in the series of functions consisting of Chebyshev polynomials. The expansion function $\phi_n(x)$ and $\psi_n(x)$ are expressed as

$$\phi_n(x) = (1 \quad x^2)C_n(x), \qquad \psi_n(x) = (1 \quad x^2)^2 C_n(x) \tag{7}$$

where $C_n(x) = \cos(n\cos^{-1}(x))$ is the n^{th} order Chebyshev polynomial. $w(x,y,t), \psi(x,y,t)$ and $T(x,y,t)$ are expanded in terms of the expansion functions $\phi_n(x)$ and $\psi_n(x)$ as:

$$w(x,y,t) = \sum_{m=0}^{M}\sum_{n=0}^{N} w_{mn}(t)\varphi_m(x)\varphi_n(y)$$
$$\psi(x,y,t) = \sum_{m=0}^{M}\sum_{n=0}^{N} \psi_{mn}(t)\psi_m(x)\psi_n(y). \tag{8}$$
$$T(x,,y,t) = \sum_{m=0}^{M}\sum_{n=0}^{N} T_{mn}\,\varphi_m(x)\varphi_n(y) + x$$

where M and N are the truncation numbers in the x and y directions respectively. Unsteady solutions are obtained by using Crank-Nicolson and Adams-Bashforth methods together with the function expansion and collocation methods.

4. RESISTANT COEFFICIENT

The resistant coefficient λ is used as the representative quantity of the flow state and is generally used in fluids engineering, defined as

$$\frac{P_1^* \quad P_{21}^*}{\Delta_{z^*}} = \frac{\lambda}{d_h^*}\frac{1}{2}\rho \quad \omega^{* \quad 2} \tag{9}$$

where quantities with an P_1^* denotes dimensional ones,

$\langle\ \rangle$ stands for the mean over the cross section of the duct and d_h^* is the hydraulic diameter. The main axial velocity $\langle \omega^* \rangle$ is calculated by

$$\langle \omega^* \rangle = \frac{v}{4\sqrt{2\delta}d} \int_{-1}^{1} dx \int_{-1}^{1} \omega(x, y, t) dy \qquad (10)$$

Since $\left(P_1^* - P_2^*\right)/\Delta_{z^*} = G$, λ is related to the mean non-dimensional axial velocity $\langle \omega \rangle$ as

$$\lambda = \frac{4\sqrt{2\delta}Dn}{\langle \omega \rangle^2} \qquad (11)$$

where $\langle \omega \rangle = \sqrt{2\delta}d \langle \omega^* \rangle / v$.

5. RESULTS AND DISCUSSION

5.1 *Case I*: Steady Solution

With the present numerical calculation, we obtain two branches of steady solutions for the curvature $\delta = 0.5$ and the Grashof number $Gr = 100$ over the Dean number $100 \le Dn \le 6500$. The two steady solution branches are named the *first steady solution branch* (first branch, bold solid line) and the *second steady solution branch* (second branch, thin solid line), respectively. It should be noted here that Mondal *et al*. (2008) obtained two branches of steady solutions for the isothermal flow through a curved square duct with small curvature. Figure 3 shows solution structure of the steady solutions for the flow through a curved square channel with strong curvature. Figure 3 shows contours of secondary flow, axial flow and temperature profile at some specific values of *Dn*.

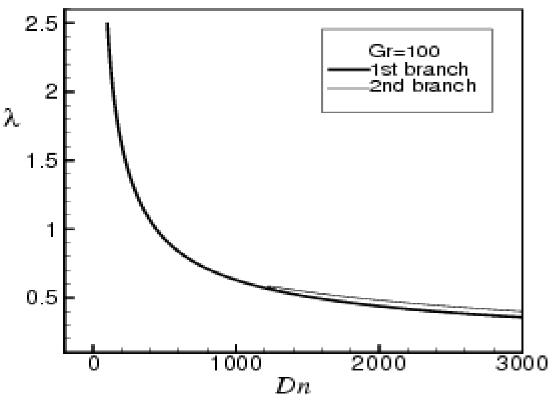


Fig 2. Steady solution branches for curvature $\delta = 0.5$ for Gr = 100

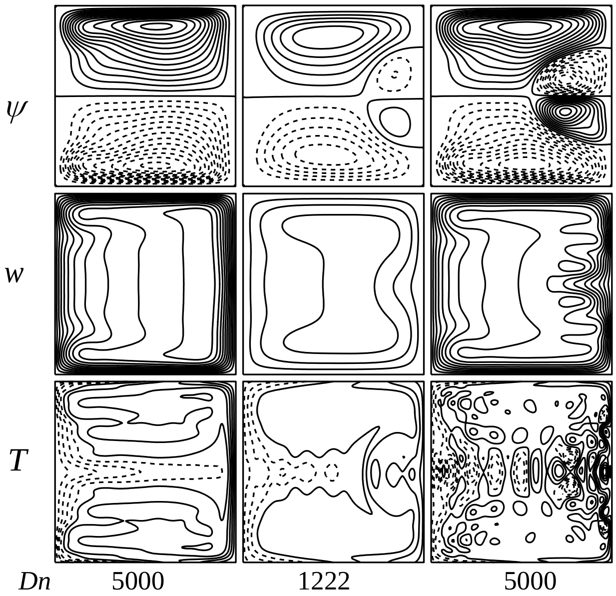
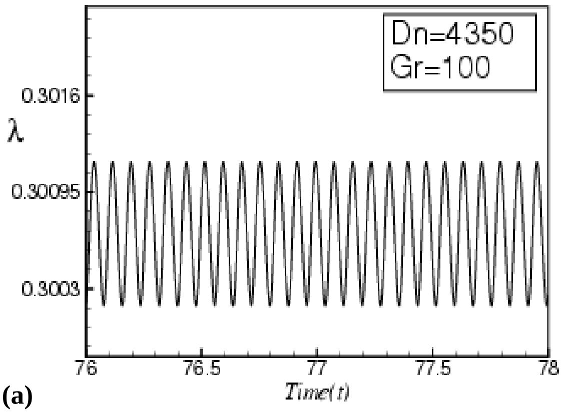


Fig 3: Secondary flow (top), axial distribution (middle) and temperature profile (below) for different *Dn* numbers.

5.2 *Case II*: Unsteady Solution

5.2.1. Time evolutions for $4350 \le Dn \le 4475$

We studied the time evolution of the resistance coefficient λ for $4350 \le Dn \le 4440$. It is found that the flow is periodic for all the values of Dn in this range. Figure 4(a) shows that the flow is periodic oscillations for Dn = 4350. In order to investigate the periodic behavior more clearly, power spectrum of the time evaluation for $Dn = 4350$ are shown in Figure 4 (b), where the line spectra of the fundamental frequency and its harmonics are seen, which suggests that the flow is periodic for this case. Typical contours of secondary flow; axial flow distribution and temperature profiles are shown in Figure 4(c) for $Dn = 4350$ for one period of oscillation at time $76.80 \le t \le 76.88$. As seen in Figures 4(c), the secondary flow is a two -vortex solutions for Dn = 4350.



(a)

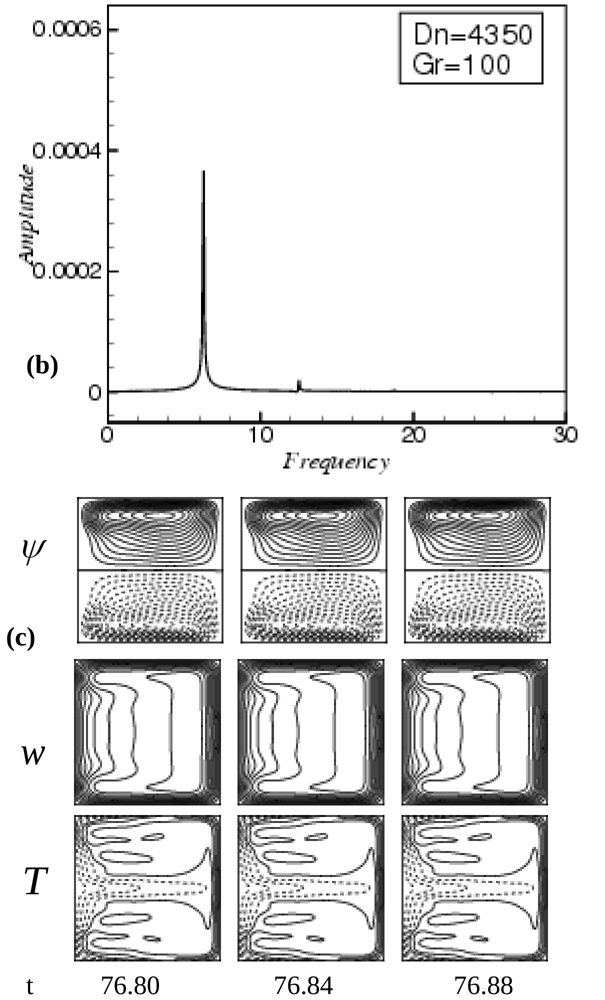


Fig 4: (a) Time evolution of λ for $Dn = 4350$ at time $76 \le t \le 78$ for $\delta = 0.5$ (b) Power spectra of the time evolution. (c) Contours of secondary flow, axial flow distribution and temperature profiles for $Dn = 4350$ at time $76.80 \le t \le 76.88$

5.2.2. Time Evolution For $5075 \le Dn \le 5835$

We perform time evolution of λ for $5075 \le Dn \le 5835$. It is found that all the flow oscillates irregularly that is the flow is chaotic in this range. Figure 5(a) show the instances of chaotic oscillations for $Dn = 5835$. In order to investigate the chaotic behavior more clearly, power spectra of the time evaluation of λ for $Dn = 5835$ are shown in Figures 5(b), where we see that lots of continuous line spectra with different frequencies are seen, this result suggests that the flow is chaotic. To observe that the change of the flow characteristics, contours of typical secondary flow patterns, axial distribution and temperature profiles are shown in Figure 5(c) for $Dn = 5835$, where it is seen that the chaotic oscillation for $Dn = 5835$ oscillates between asymmetric two-vortex solutions.

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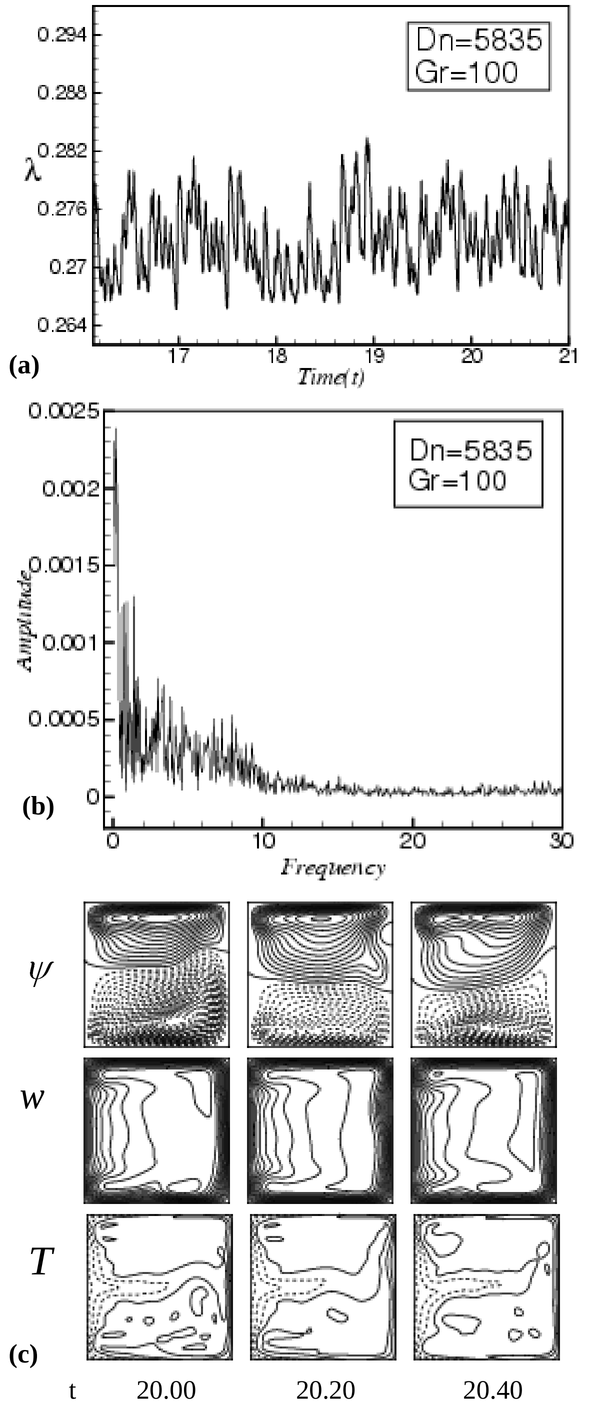
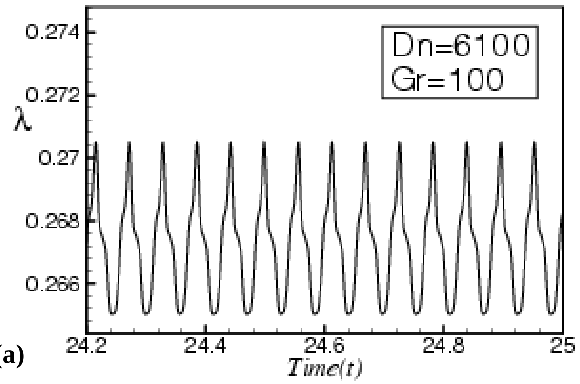


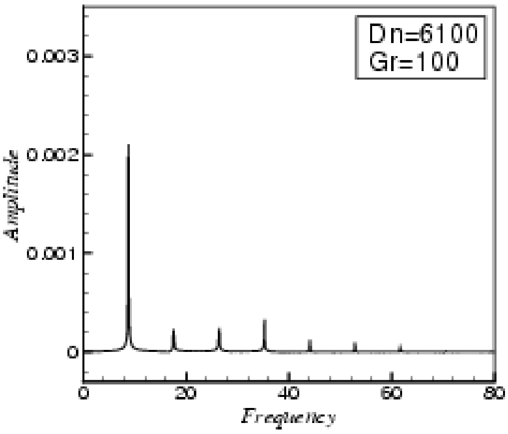
Fig 5: (a) Time evolution of λ for $Dn = 5835$. (b) Power spectra of the time evolution of λ for $Dn = 5835$. (c) Contours of secondary flow, axial flow distribution and temperature profiles for $Dn = 5835$.

5.2.3 Time Evolution For $6080 \le Dn \le 6100$

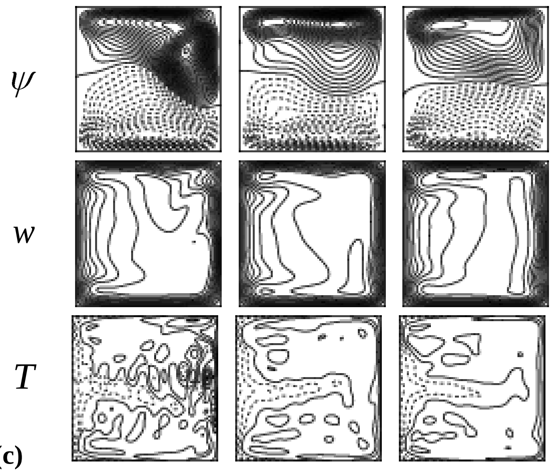
We studied the time evolution of the resistance coefficient λ for $6080 \le Dn \le 6100$. It is found that the flow is multi-periodic for all the values of Dn in this range. In order to investigate the multi-periodic behavior more clearly, power spectrum of the time evaluation for $Dn = 6100$ are shown in Figure 6(b), where the line spectra of the fundamental frequency and the frequencies are harmonic which suggests that the flow are multi-periodic. To observe that the change of the flow characteristics, contours of typical secondary flow patterns, axial distribution and temperature profiles are shown in Figure 6(c) for $Dn = 6100$, where it is seen that the multi-periodic oscillation for $Dn = 6100$ oscillates two- vortex solutions.



(a)



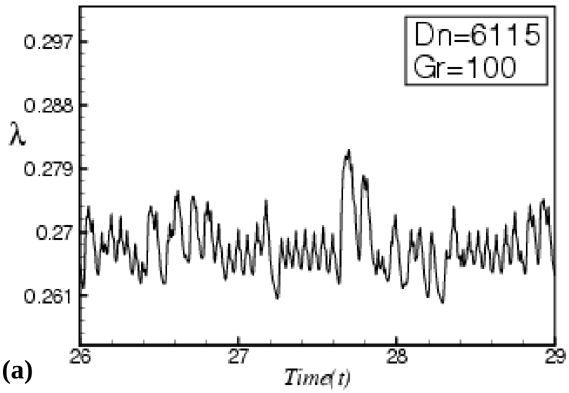
(b)



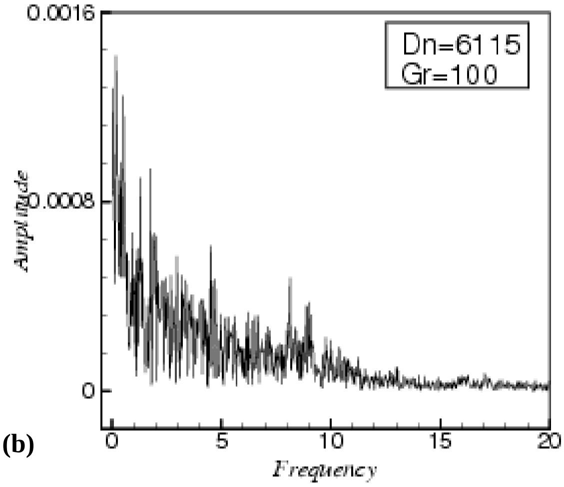
(c)

t 24.65 24.69 24.71

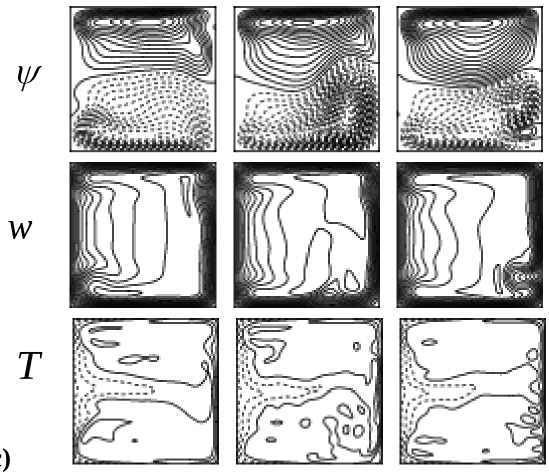
Fig 6. (a) Time evolution for $Dn = 6100$ (b) Power spectra of λ, (c) Contours of secondary flow, axial flow distribution and temperature profiles for $Dn = 6100$



(a)



(b)



(c)

t 28.00 28.20 28.40

Fig 7. (a) Time evolution of λ for $Dn = 6115$ for $\delta = 0.5$ (b) Power spectra of the time evolution of λ for $Dn = 6115$ (c) Contours of secondary flow, axial flow and temperature profiles for $Dn = 6115$

5.2.4. Time Evolution For $6115 \le Dn \le 6500$

We increase the Dean number and investigate time evolution of λ for $6115 \le Dn \le 6500$. It is found that all the flows oscillate Chaotic in this range. Figure 7(a) show the instances of chaotic oscillations for $Dn = 6115$. In order to investigate the chaotic behavior more clearly, power spectra of the time evaluation of λ for $Dn = 6115$ are shown in Figure 7(b), we see that lots of continuous line spectra with different frequencies are seen, this result suggests that the flow is chaotic. To observe that the change of the flow characteristics contours of typical secondary flow patterns, axial distribution and temperature profiles are shown in Figure 7(c)), where it is seen that the Chaotic oscillation for $Dn = 6115$ oscillates between asymmetric two-,three-and four-vortex solutions three-vortex solutions.

6. CONCLUSIONS

In this study, a comprehensive numerical result is presented for the flow through a curved square channel with curvature 0.5 over a wide range of the Dean number for the Grashof number $Gr = 100$. Spectral method is used as a basic tool to solve the system non-linear differential equations. We obtained two branches of asymmetric steady solutions with two- and four-vortex solutions. It is found that the first branch consists two vortex solutions, while the second branch two- and four-vortex solutions. In the unstable region, time-evolution calculations of the unsteady solutions are performed. Time evaluations calculations as well as their spectra analyses show that the steady flow turns into chaos flow through periodic or multi-periodic oscillations in the straightforward scenario steady-periodic –chaotic , If Dn is increased. It is found that if the Dean number is increased, the temperature from heated wall to the fluid passed very significantly. Axial flow distribution is consistent with the secondary vortices. It is found that the transition to periodic or chaotic state is retarded consistently as the curvature is increased. In order to investigate the transition from the multi-periodic oscillations for the chaotic states in more detail, the spectral analysis is found to be very useful. In this regard, it is interesting to notice that the chaotic solution difference in a small Dean number. When there is no stable steady solutions, time evolution of λ is obtained and it is found in the unstable region the flow undergoes through various flow instabilities, if Dn is increased.

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